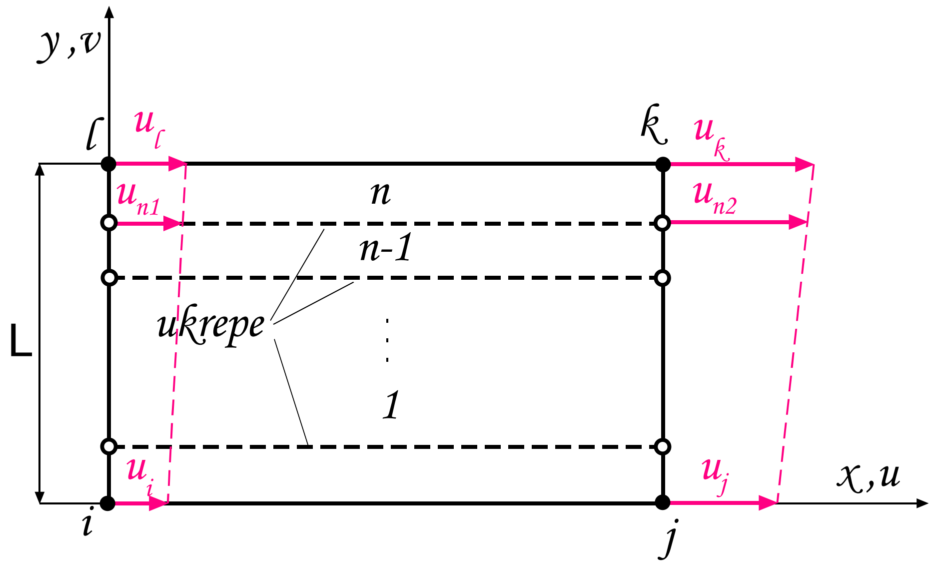
Makroelement orebrene membrane



P –oplata (*plating*, membrana)
S – ukrepa (*stiffener*, štap)
Unutrašnja energija je definirana kao:

$$\mathbf{U}^{eSP} = \mathbf{U}^{P} + \mathbf{U}^{S} = \frac{1}{2}\mathbf{u}^{T}\mathbf{K}^{eSP}\mathbf{u} = \frac{1}{2}\mathbf{u}^{T}\mathbf{K}^{P}\mathbf{u} + \frac{1}{2}\mathbf{u}^{T}\mathbf{K}^{S}\mathbf{u} \quad (1)$$

Ukupna matrica krutosti elementa

$$\mathbf{K}^{eSP} = \mathbf{K}^{P} + \mathbf{K}^{S} \quad (2)$$

Matrica krutosti štapova koji ukrepljuju membranu:

$$\mathbf{K}^{S} = \sum_{n=1}^{N}\mathbf{K}^{S}_{Gn} \quad (3)$$

gdje je:

$$\mathbf{K}^{S}_{Gn} = \mathbf{T}^{T}\mathbf{K}^{S}_{Ln}\mathbf{T} \quad (4)$$
- matrica krutosti jedne ukrepe u globalnom koo. sustavu

$$\mathbf{K}^{S}_{Ln} = \frac{A_n E_n}{L_n}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (5)$$
- matrica krutosti jedne ukrepe u lokalnom koo. sustavu

T - matrica transformacije
N - ukupni broj ukrepa

Lokalni pomak internog čvora 1 na ukrepi *n*:

$$u_{n1} = \mathbf{N}_1^{T}\mathbf{u} = \begin{Bmatrix} 1-\dfrac{y_n}{L} & \dfrac{y_n}{L} \end{Bmatrix}\begin{Bmatrix} u_i \\ u_l \end{Bmatrix} \quad (6)$$

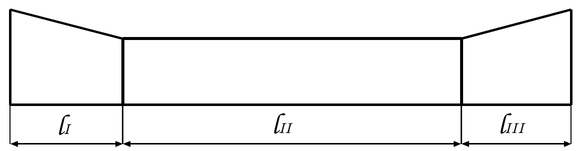
Lokalni pomaci ukrepe *n* (štapnog elementa)

$$\mathbf{u}_{Ln} = \begin{Bmatrix} u_{n1} \\ u_{n2} \end{Bmatrix} = \begin{Bmatrix} 1-\frac{y_n}{L} & 0 & 0 & \frac{y_n}{L} \\ 0 & 1-\frac{y_n}{L} & \frac{y_n}{L} & 0 \end{Bmatrix} \begin{Bmatrix} u_i \\ u_j \\ u_k \\ u_l \end{Bmatrix} = \mathbf{T} \cdot \mathbf{u}_G \tag{7}$$

Uz poznatu matricu transformacije **T** moguće je zapisati ukupnu matricu krutosti makroelementa orebrene membrane kao:

$$\mathbf{K}^{eSP} = \mathbf{K}^{P} + \sum_{n=1}^{N} \mathbf{T}_n^T \mathbf{K}_{Ln}^S \mathbf{T}_n \tag{8}$$

Makroelement grede s koljenima



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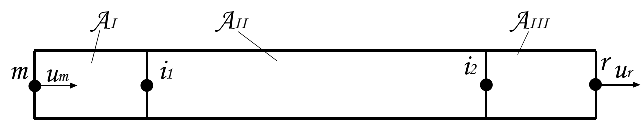
① štap



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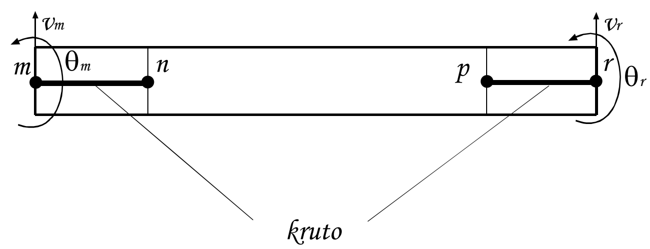
② greda



1) Štap

Matrice krutosti pojedinih elemenata

$$\mathbf{k_I} = \frac{A_I E_I}{l_I}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix};\ \mathbf{k_{II}} = \frac{A_{II} E_{II}}{l_{II}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix};\ \mathbf{k_{III}} = \frac{A_{III} E_{III}}{l_{III}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{k_I} = \begin{bmatrix} k_{I_{11}} & k_{I_{12}} \\ k_{I_{21}} & k_{I_{22}} \end{bmatrix}; \mathbf{k_{II}} = \begin{bmatrix} k_{II_{11}} & k_{II_{12}} \\ k_{II_{21}} & k_{II_{22}} \end{bmatrix}; \mathbf{k_{III}} = \begin{bmatrix} k_{III_{11}} & k_{III_{12}} \\ k_{III_{21}} & k_{III_{22}} \end{bmatrix}$$

Slaganje matrice krutosti s internim i eksternim čvorovima

	i_1	i_2	m	r
i_1	$k_{I_{22}} + k_{II_{11}}$	$k_{II_{12}}$	$k_{I_{21}}$	
i_2	$k_{II_{21}}$	$k_{II_{22}} + k_{III_{11}}$		$k_{III_{12}}$
m	$k_{I_{12}}$		$k_{I_{11}}$	
r		$k_{III_{21}}$		$k_{III_{22}}$

$$\left[\begin{array}{cc|cc} k_{I_{22}} + k_{II_{11}} & k_{II_{12}} & k_{I_{21}} & \\ k_{II_{21}} & k_{II_{22}} + k_{III_{11}} & & k_{III_{12}} \\ \hline k_{I_{12}} & & k_{I_{11}} & \\ & k_{III_{21}} & & k_{III_{22}} \end{array}\right] \cdot \begin{Bmatrix} u_1 \\ u_2 \\ \hline u_m \\ u_r \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \hline f_m \\ f_r \end{Bmatrix}$$

← interni

← eksterni

$$\begin{bmatrix} \mathbf{k_{ii}} & \mathbf{k_{ie}} \\ \mathbf{k_{ie}^T} & \mathbf{k_{ee}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_i} \\ \mathbf{u_e} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f_e} \end{Bmatrix}$$

Ne interesiraju nas pomaci internih čvorova pa ih se trebamo riješiti.

$$\mathbf{k_{ii}} \cdot \mathbf{u_i} + \mathbf{k_{ie}} \cdot \mathbf{u_e} = \mathbf{0}$$
$$\mathbf{k_{ie}^T} \cdot \mathbf{u_i} + \mathbf{k_{ee}} \cdot \mathbf{u_e} = \mathbf{f_e}$$

Iz prve jednadžbe može se izlučiti $\mathbf{u_i}$:

$$\mathbf{k_{ii}^{-1}} \cdot \Big| \quad \mathbf{k_{ii}} \cdot \mathbf{u_i} = -\mathbf{k_{ie}} \cdot \mathbf{u_e}$$
$$\mathbf{u_i} = -\mathbf{k_{ii}^{-1}} \cdot \mathbf{k_{ie}} \cdot \mathbf{u_e}$$

Uvrštavanjem u drugu jednadžbu dobivamo

$$\mathbf{k_{ie}^T} \cdot \left(-\mathbf{k_{ii}^{-1}} \cdot \mathbf{k_{ie}} \cdot \mathbf{u_e}\right) + \mathbf{k_{ee}} \cdot \mathbf{u_e} = \mathbf{f_e}$$
$$\left(-\mathbf{k_{ie}^T} \cdot \mathbf{k_{ii}^{-1}} \cdot \mathbf{k_{ie}} + \mathbf{k_{ee}}\right)\mathbf{u_e} = \mathbf{f_e}$$

odnosno:

$$\mathbf{k_u} \cdot \mathbf{u_e} = \mathbf{f_e}$$
$$\mathbf{k_u} = -\mathbf{k_{ie}^T} \cdot \mathbf{k_{ii}^{-1}} \cdot \mathbf{k_{ie}} + \mathbf{k_{ee}}$$

2) Greda

$$\left.\begin{aligned} \theta_n &= \theta_m \\ v_n &= v_m + l_m \cdot \theta_m \end{aligned}\right\} \text{ - vrijedi i na drugoj strani}$$

Lokalni pomaci:

$$\mathbf{v_L} = \left\{ v_n \quad \theta_n \quad v_p \quad \theta_p \right\}$$

Globalni pomaci:

$$\mathbf{v_G} = \left\{ v_m \quad \theta_m \quad v_r \quad \theta_r \right\}$$

Unutrašnja energija:

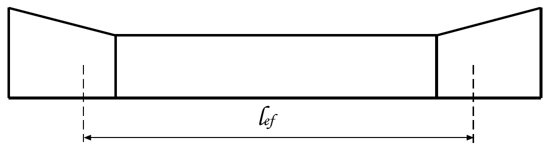
$$U_L = \frac{1}{2} \mathbf{v_L^T} \cdot \mathbf{k_B} \cdot \mathbf{v_L}$$

$$\begin{Bmatrix} v_n \\ \theta_n \\ v_p \\ \theta_p \end{Bmatrix} = \begin{bmatrix} 1 & l_m & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & lr \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} v_m \\ \theta_m \\ v_r \\ \theta_r \end{Bmatrix} = \mathbf{T} \cdot \mathbf{v_G}$$

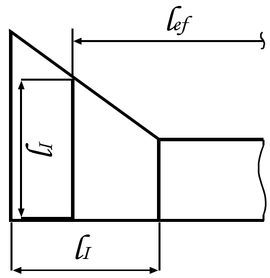
$U_L = U_G$ - (zato što kruti krajevi ne apsorbiraju energiju)

$$U_G = \frac{1}{2} \mathbf{v_G^T} \cdot \underbrace{\mathbf{T}^T \cdot \mathbf{k_B} \cdot \mathbf{T}}_{\mathbf{k_{BG}}} \cdot \mathbf{v_G}$$

$\mathbf{k_{BG}}$ - globalna matica krutosti grede s krutim krajevima



efektivnu duljinu propisuju
klasifikacijska drustva



Uobicajena pretpostavka

Ako se pri modeliranju ne uzmu u obzir kruti krajevi, onda bi analiza mogla pokazati da konstrukcija puca iako se to u biti ne događa. To znači da je model na strani sigurnosti, no posljedica veća težina i cijena.

Proces kojim su se od 4 čvora dobila dva lokalna naziva se **statička kondenzacija**.
Napomene pri modeliranju:

- teorija grede vrijedi do pomaka od 2 odnosno maksimalno 3 promila duljine grede
- izvan tog područja potrebno je koristiti nelinearnu analizu
- uže na jarbolu se ponaša nelinearno u konstrukciji jer na tlak uopće ne nosi dok se pri vlačnim opterećenjima ponaša linearno